CP VIOLATION

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We review the present status of the Standard Model of CP violation, which is based on a complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. So far CP violation has been observed only in $K^0 - \overline{K}^0$ mixing, consistent with a sizable phase. The implications of future CP nonconserving measurements in K and B decays are discussed within the model. Direct CP violation in $K \to 2\pi$ may be observed in the near future, however this would not be a powerful test of the model. B decays provide a wide variety of CP asymmetry measurements, which can serve as precise tests of the Standard Model in cases where the asymmetry is cleanly related to phases of CKM matrix elements. Some of the most promising cases are discussed.

1. INTRODUCTION

The history of the subject of CP violation is thirty years old. The first few tens of $K_L \to \pi^+\pi^-$ events observed by Christenson, Cronin, Fitch and Turlay [1] led the way to millions of events, in which long-lived neutral kaons were observed to decay to both charged and neutral pions. So far the neutral kaon system remains the only system in which CP nonconservation has been measured. For three decades continuously improving K decay experiments verified the single fact that CP is violated in $K^0 - \overline{K}^0$ mixing [2]. The parameter which describes this phenomenon is ϵ, the tiny CP impurity in the short- and long-lived kaon states. Its present world-average value [3], $|\epsilon| = (2.26 \pm 0.02) \times 10^{-3}$, is measured by $|\eta_{+-}| = |\eta_{00}|$, the K_L to K_S ratio of decay amplitudes into charged and neutral two pion states. The independent measurement of $2\mathrm{Re}\epsilon$ by the charge asymmetry in $K_L \to \pi\ell\nu$ [2] is consistent with this value and with the phase measurement of η_{+-}.

The search for direct CP violation in the $K \to \pi\pi$ decay process was the main purpose of two experiments operating for a lengthly period starting in 1986 at Fermilab and at CERN. The results of these experiments, looking for a difference between η_{+-} and η_{00}, were $\mathrm{Re}(\epsilon'/\epsilon) = (7.4 \pm 5.9) \times 10^{-4}$ (Fermilab E731 [4]) and $(23 \pm 6.5) \times 10^{-4}$ (CERN NA31 [5]). This did not provide unambiguous evidence for a nonzero effect.

CPT is a cherished symmetry of quantum field theory. It is quite important to establish

a high precision for this symmetry in the K system, where it implies an equivalence between CP violation and the breaking of time reversal symmetry. One such test [6], $\mathrm{Arg}(\eta_{+-}) = \mathrm{Arg}(\eta_{00}) = \tan^{-1}(-2\Delta m/\Delta\Gamma)$ (where ΔM and $\Delta\Gamma$ are the neutral K meson mass- and width-differences), suffered for a while from a two standard deviation discrepancy [7]. Recent experiments at Fermilab and at the Low Energy Antiproton Ring (LEAR) at CERN have, however, measured somewhat smaller values for Δm and for the phase of η_{+-}, in very good agreement with this relation [8].

Future plans of experimental CP studies in strange particle decays [9] include searching for direct CP violation in $K \to \pi\pi$, at CERN, at Fermilab and at the Frascatti Φ factory. It is hoped to reach by 1996 a level of 10^{-4} in $\mathrm{Re}(\epsilon'/\epsilon)$ and to improve the present precision of the CPT test. Other decay processes, in which CP violation effects will be looked for, include: $K \to 3\pi$, $K_L \to \pi^0 e^+ e^-$, $K_L \to \pi^0 \nu\overline{\nu}$ and hyperon decays.

It seems, however, that the next decade of CP violation studies will be dominated by the B meson system. Very useful information, such as the large $B^0 - \overline{B}^0$ mixing, measurements of relevant branching ratios, and feasibility studies of rare decays which are important for CP violation, was already obtained from Doris II at DESY and from presently operating facilities, mostly from CESR at Cornell, the LEP accelerator at CERN and the Fermilab Tevatron. It is not entirely unlikely that in the near future experiments at these accelerators will provide us with first serious studies of CP violation in the B system. In about four years one expects the starting operation of three large scale experiments dedicated to thi purpose, at two asymmetric e^+e^- B factories at SLAC and at KEK, and at HERA using an internal target at the proton ring. By the end of this millennium these facilities can provide measurements of CP asymmetries in a few B decay channels. Further out in the future one may expect that the LHC at CERN will provide us with the ultimate abundant production of B mesons, where a special B-physics program has the potential of high precision CP studies.

The Standard Model [10] provides a suitable framework for understanding the CP violation observed in the neutral K meson mixing. The single source of CP violation in the theory is a phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix [11]. The only information about this phase comes from the measured value of the CP impurity mixing parameter ϵ. Thus, while this single measurement can be accommodated in the CKM theory, it cannot test the theory. The predictions of direct CP violation in strangeness-changing processes, such as $K \to \pi\pi$ and other K and hyperon decays involve large theoretical uncertainties. These measurements are important for their own sake, just to demonstrate CP violation outside $K^0 - \overline{K}^0$ mixing, however due to theoretical uncertainties they cannot serve as powerful tests of the Standard Model. On the other hand, the B meson system provides a wide variety of

independent CP asymmetry measurements related to different sectors of the CKM matrix. Some of these asymmetries can be related to corresponding CKM phases in manner which is free of theoretical uncertainties. These phases are fundamental parameters of the Standard Model, just as the electron or the t-quark mass. Thus, it seems that CP violation in B decays is due to become a fertile ground for testing the CKM symmetry breaking mechanism.

In this theoretical review we discuss CP violation in the K and B meson systems within the Standard Model. We begin in Section 2 by introducing the CKM matrix. We summarize the available information on the magnitude of its elements and on their CP violating phases. The general formalism of CP nonconservation in neutral meson mixing is described in Section 3, where we note an important difference between $K^0 - \overline{K}^0$ and $B^0 - \overline{B}^0$ mixing. Section 4 treats the K meson system. We discuss our present theoretical understanding of ϵ, the measured CP impurity in the neutral K meson system, and the theoretical uncertainties of calculating direct CP violation in $K \to \pi\pi$. CP violation in the B meson system is studied in Section 5. We distinguish between three kinds of CP nonconserving phenomena and discuss in turn their predictions within the Standard Model: 1. CP violation in $B^0 - \overline{B}^0$ mixing; 2. CP violation which occurs when mixed neutral B mesons decay to states which are common decay products of B^0 and $\overline{B}^0$; 3. Direct CP violation in charged B decays. Our focus is mainly on CP asymmetries which can be related to fundamental CKM phases in a manner which involves no, or very small, theoretical uncertainties. We describes two different methods of neutral B flavor-tagging, which is needed for asymmetry measurements in neutral B decays. Section 6 summarizes with a few concluding remarks.

This review is not supposed to be complete. Complementary discussions with further references can be found in several previous reviews [12]. As mentioned, we will only consider CP violation with K and B mesons. Other related topics, such as the limit on the neutron electric dipole moment, the strong CP problem and the baryon asymmetry in the universe , will not be dealt with due to shortage of time and since they do not seem to have direct consequences in the Standard Model. We will restrict our discussion to CP violating predictions within the CKM Model. Studies of alternative possible mechanisms of CP violation, with corresponding predictions, can be found elsewhere [12].

2. CP VIOLATION IN THE STANDARD MODEL: THE CKM MATRIX

In the standard model the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group is spontaneously broken by the vacuum expectation value of a single scalar Higgs doublet. CP violation occurs in the interactions of the three families of left-handed quarks with the charged gauge

boson:

$$-\mathcal{L} = (\,\overline{u} \quad \overline{c} \quad \overline{t}\,) \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix} + (\,\overline{d} \quad \overline{s} \quad \overline{b}\,) \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$+\frac{g}{\sqrt{2}} (\,\overline{u} \quad \overline{c} \quad \overline{t}\,)_L \gamma^\mu V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W^+_\mu + ... \tag{1}$$

CP violation requires a complex Cabibbo-Kobayashi-Maskawa [11] (CKM) mixing matrix V. The quark mass terms exhibit a symmetry under phase redefinitions of the six quark fields. This freedom leaves a single phase in V. The unitary matrix V, which can be defined in terms of this phase (γ) and three Euler-like mixing angles, is approximated for most practical purposes by the following form:

$$V \approx \begin{pmatrix} 1 & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{us}| & 1 & |V_{cb}| \\ |V_{us}V_{cb}| - |V_{ub}|e^{i\gamma} & -|V_{cb}| & 1 \end{pmatrix} . \tag{2}$$

The measured values of the three mixing angles ($\sin\theta_{12} \equiv |V_{us}|,\ \sin\theta_{23} \equiv |V_{cb}|,\ \sin\theta_{13} \equiv |V_{ub}|$) have a hierarchical pattern in generation space [13],

$$|V_{us}| = 0.220 \pm 0.002 \ (\lambda) \ ,$$

$$|V_{cb}| = 0.038 \pm 0.005 \ (\mathcal{O}(\lambda^2)) \ ,$$

$$|V_{ub}| = 0.0035 \pm 0.0015 \ (\mathcal{O}(\lambda^3)) \ , \tag{3}$$

often characterized by powers of a parameter $\lambda \equiv \sin\theta_c = 0.22$ [14]. This structure was used with unitarity to obtain the approximate expressions of the three t quark couplings in V. It is amusing to note that the yet unmeasured value of $|V_{tb}|$ obtained from unitarity is the most accurately known parameter of the mixing matrix.

Unitarity of V can be represented geometrically in terms of triangles, such as the one depicted in Fig. 1 representing the relation

$$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0 \ . \tag{4}$$

Figure 1: The CKM unitarity triangle

The three angles of the unitarity triangle, α, β and γ (which appears as a phase in (2)), are rather badly known at present. Current constarints, which depend on uncertainties in K- and B-meson hadronic parameters, can be approximately summarized by the following ranges [15]:

$$10^0 \le \alpha \le 150^0, \quad 5^0 \le \beta \le 45^0, \quad 20^0 \le \gamma \le 165^0 \ . \tag{5}$$

As we will show, certain CP asymmetries in B decays are directly related to these angles in a manner which is free of hadronic uncertainties, and can provide a more precise determination for some of these fundamental parameters.

One can draw similar unitarity triangles describing the orthogonality of other pairs of columns or rows of the CKM matrix. Knowledge of the angles of all these triangles, which can be related to CP asymmetries, suffice to determine the entire matrix [16]. All such triangles have equal areas, however they involve one side which is much shorter than the other two sides, and consequently one of their angles is very tiny. This is in contrast to the angles α, β and γ which are naturally large, since all the three sides of the unitarity triangle of Fig. 1 are of comparable ($\mathcal{O}(\lambda^3)$) magnitude. Thus, for instance, the neutral K meson triangle, built of elements $V_{qd}V^*_{qs}$ ($q = u, c, t$), has two long sides (length λ) and one extremely short side (length $\mathcal{O}(\lambda^5)$). This explains why CP asymmetries in K decays, which are related to the tiny angle of this triangle ($\mathcal{O}(\lambda^4)$), are of order 10^{-3}.

3. CP VIOLATION IN NEUTRAL MESON MIXING

The flavor states P^0 and $\overline{P}^0$ (P can be either a K or a B pseudoscalar meson) mix through the weak interactions to form the "Light" and "Heavy" mass-eigenstates P_L and P_H:

$$\begin{aligned} |P_L\rangle &= p|P^0\rangle + q|\overline{P}^0\rangle \ , \\ |P_H\rangle &= p|P^0\rangle - q|\overline{P}^0\rangle \ . \end{aligned} \tag{6}$$

These states have masses $m_{L,H}$ and widths $\Gamma_{L,H}$. The Hamiltonian eigenvalue equation (using CPT)

$$\begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M^*_{12} - \frac{i}{2}\Gamma^*_{12} & M - \frac{i}{2}\Gamma \end{pmatrix} \begin{pmatrix} p \\ \pm q \end{pmatrix} = (m_{L,H} - \frac{i}{2}\Gamma_{L,H}) \begin{pmatrix} p \\ \pm q \end{pmatrix} \tag{7}$$

has the following solution for the mixing parameter $q/p \equiv (1-\tilde{\epsilon})/(1+\tilde{\epsilon})$:

$$\frac{q}{p} = \sqrt{\frac{M^*_{12} - \frac{i}{2}\Gamma^*_{12}}{M_{12} - \frac{i}{2}\Gamma_{12}}} = -\frac{2(M^*_{12} - \frac{i}{2}\Gamma^*_{12})}{\Delta m - \frac{i}{2}\Delta\Gamma} \ , \tag{8}$$

where $\Delta m \equiv m_H - m_L, \Delta\Gamma \equiv \Gamma_H - \Gamma_L$. M_{12} and Γ_{12} describe respectively transitions from P^0 to $\overline{P}^0$ via virtual states and contributions from decay channels which are common to P^0 and $\overline{P}^0$.

The CP impurity parameter $\tilde{\epsilon}$ gives the mass-eigenstates in terms of states with well-defined CP

$$
\begin{aligned}
|P_L\rangle &= \frac{1}{\sqrt{1+|\tilde{\epsilon}|^2}}(|P_1^0\rangle + \tilde{\epsilon}|P_2^0\rangle) , \\
|P_H\rangle &= \frac{1}{\sqrt{1+|\tilde{\epsilon}|^2}}(|P_2^0\rangle + \tilde{\epsilon}|P_1^0\rangle) , \\
|P_1\rangle &= \frac{1}{\sqrt{2}}(|P^0\rangle + |\overline{P}^0\rangle) , \\
|P_2\rangle &= \frac{1}{\sqrt{2}}(|P^0\rangle - |\overline{P}^0\rangle) .
\end{aligned} \tag{9}
$$

q/p has a phase freedom under redefinition of the phases of the flavor states P^0, $\overline{P}^0$: $|P^0\rangle \to e^{i\xi}|P^0\rangle$, $|\overline{P}^0\rangle \to e^{-i\xi}|\overline{P}^0\rangle \;\Rightarrow\; (q/p) \to e^{2i\xi}(q/p)$. Thus the phase of q/p can be rotated away and $|q/p| = 1$ means CP conservation in $P^0 - \overline{P}^0$ mixing. The deviation of $|q/p|$ from one measures CP violation in the mixing:

$$
1 - |\frac{q}{p}| \approx 2\mathrm{Re}\tilde{\epsilon} , \tag{10}
$$

where $\mathrm{Re}\tilde{\epsilon}$ is phase-convention independent. For convenience, we will use the quark phase convention in which the CKM matrix (2) is written.

It is clear from eq.(8) that CP violation in neutral meson mixing is expected to be small under two different circumstances:

$$
\begin{aligned}
\mathrm{Arg}M_{12} &\approx \mathrm{Arg}(-\Gamma_{12}) \quad (K \text{ meson}) , \\
|\Gamma_{12}| &\ll |M_{12}| \quad (B \text{ meson}) .
\end{aligned} \tag{11}
$$

The first case applies to the neutral K meson system and the second one - to B mesons. The different circumstances allude to the reason for the small and theoretically uncertain CP violation in K decays in constrast to the large and theoretically clean CP violation in B decays. In K decays Γ_{12} is dominated by the 2π channel, the amplitude of which involves (in the CKM phase convention) a very small phase which is even smaller than the small phase of M_{12}. The calculation of both phases involve hadronic uncertainties. On the other hand, the second condition, which applies to the neutral B meson system, says nothing about phases of decay amplitudes which can be and in fact are large. The phase of q/p, which can be approximated by the phase of M_{12}^*, helps in relating the expected large CP asymmetries to pure CKM parameters.

4. THE K MESON SYSTEM

4.1 CP Violation in $K^0 - \overline{K}^0$ Mixing

In the CKM phase convention M_{12} obtaines a small imaginary contribution from t and c quarks in the box-diagrams of Fig. 2, and Γ_{12} has a much smaller imaginary part from $K \to 2\pi$ (see Sec. 4.2).

Figure 2: Box diagrams for $\mathrm{Im}M_{12}(K)$

Thus we have

$$
\begin{aligned}
2|M_{12}| &= \Delta m_K \equiv m_L - m_S \ , \\
2|\Gamma_{12}| &= -\Delta\Gamma_K \equiv \Gamma_S - \Gamma_L \ ,
\end{aligned}
\tag{12}
$$

where we used the conventional notations for the long- and short-lived kaons. We find

$$
\tilde{\epsilon}_K \approx \frac{i\mathrm{Im}M_{12}}{\Delta m_K - \frac{i}{2}\Delta\Gamma_K} = \frac{\mathrm{Im}M_{12}}{\sqrt{2}\Delta m_K} e^{i\phi_K} \ , \tag{13}
$$

where $\tan\phi_K \equiv -2\Delta m_K/\Delta\Gamma_K$, $\phi_K = (43.6 \pm 0.2)^0$ [3] $\mathrm{Re}\tilde{\epsilon}$ (which is phase-convention independent) is measured in $K \to 2\pi$ and by the charge asymmetry in $K_L \to \pi\ell\nu$. We note that in the CKM phase convention also the imaginary part of $\tilde{\epsilon}$ is given, to a good approximation, by $\mathrm{Im}\epsilon$ as measured in $K \to 2\pi$ (see Sec. 4.2).

The calculation of $\mathrm{Im}M_{12}$ uses Fig. 2 with QCD corrections to obtain the following expression for $\tilde{\epsilon}$ [17]:

$$
|\tilde{\epsilon}| \approx 60 B_K f(m_t, m_c, \eta_q, S_{ij})(S_{12}S_{23}S_{13}) \sin\gamma \ . \tag{14}
$$

The numerical coefficient includes factors such as $\pi^2, G_F^2, m_W^2, \Delta m_K/m_K$ and f_K^2. B_K gives the hadronic matrix element of the box diagram in terms of the vacuum insertion value. A possible range of values for this parameter is probably $B_K = 0.8 \pm 0.2$ [15]. The function

f [17] involves the c and t quark masses, calculable QCD correction factors η_q and the quark mixing angles, $S_{12} \equiv |V_{us}|$, $S_{23} \equiv |V_{cb}|$, $S_{13} \equiv |V_{ub}|$. The value of f for allowed parameters can range from 1 to 5. Finally, as any CP violating quantity, $|\tilde{\epsilon}|$ is proportinal to twice the area of the unitarity triangle, $S_{12}S_{23}S_{13}\sin\gamma$, which can obtain values in the range $(1.5 - 4.5) \times 10^{-5}\sin\gamma$. We see that the experimental value $|\tilde{\epsilon}| = 2.26 \times 10^{-3}$ can be naturally obtained for a sizable phase i the range $\sin\gamma \sim 0.1 - 1$. The prediction for $|\tilde{\epsilon}|$ includes, aside from present uncertainties in CKM parameters, also theoretical uncertainties in hadronic matrix elements and (perhaps to a less degree) - uncertainties in quark mass values and in QCD effects.

4.2 Direct CP Violation in $K \to 2\pi$

The weak amplitudes of neutral K mesons to charged and to neutral two pion states can be decomposed into amplitudes of final states with isospin $I = 0, 2$:

$$
\begin{aligned}
\langle\pi^+\pi^-|H_W|K^0\rangle &= \sqrt{\frac{2}{3}}A_0e^{i\delta_0} + \sqrt{\frac{1}{3}}A_2e^{i\delta_2} \ , \\
\langle\pi^0\pi^0|H_W|K^0\rangle &= -\sqrt{\frac{1}{3}}A_0e^{i\delta_0} + \sqrt{\frac{2}{3}}A_2e^{i\delta_2} \ , \\
\langle\pi^+\pi^-|H_W|\overline{K}^0\rangle &= \sqrt{\frac{2}{3}}A_0^*e^{i\delta_0} + \sqrt{\frac{1}{3}}A_2^*e^{i\delta_2} \ , \\
\langle\pi^0\pi^0|H_W|\overline{K}^0\rangle &= -\sqrt{\frac{1}{3}}A_0^*e^{i\delta_0} + \sqrt{\frac{2}{3}}A_2^*e^{i\delta_2} \ .
\end{aligned} \tag{15}
$$

δ_I is the elastic phase shift for $\pi\pi$ scattering at the kaon mass in an isospin I channel. A_I involves a weak CKM phase ϕ_I, which changes sign under charge-conjugation, $A_I = |A_I|e^{i\phi_I}$. Defining

$$
\eta_{+-} \equiv \frac{\langle\pi^+\pi^-|H_W|K_L\rangle}{\langle\pi^+\pi^-|H_W|K_S\rangle} \ , \quad \eta_{00} \equiv \frac{\langle\pi^0\pi^0|H_W|K_L\rangle}{\langle\pi^0\pi^0|H_W|K_S\rangle} \ ,
$$

one finds

$$
\eta_{+-} = \epsilon + \epsilon' \ , \qquad \eta_{00} = \epsilon - 2\epsilon' \ , \tag{16}
$$

where

$$
\begin{aligned}
\epsilon &= \tilde{\epsilon} + i\tan\phi_0 \ , \\
\epsilon' &= \frac{w}{\sqrt{2}}(\tan\phi_2 - \tan\phi_0)e^{i(\delta_2-\delta_0+\frac{\pi}{2})} \ .
\end{aligned} \tag{17}
$$

It is not difficult to see that Reϵ' measures CP violation in direct $K \to 2\pi$ decays. That is, it gives the rate asymmetry between the instantaneous K^0 and $\overline{K}^0$ decay widths into 2π.

ϵ and ϵ' are phase-convention independent. The equality Reϵ = Re$\tilde{\epsilon}$ does not depend on phase-convention, and Im$\epsilon \approx$ Im$\tilde{\epsilon}$ holds in the CKM phase convention, where as we

shall see $\tan\phi_0 \ll |\epsilon|$. ϵ' is suppressed by the measured $\Delta I = 3/2$ to $1/2$ suppression factor $w \equiv \mathrm{Re}A_2/\mathrm{Re}A_0 = 0.045$ [3], aside from involving the phases $\phi_0,\ \phi_2$ which are by themselves much smaller than $|\epsilon|$. This is basically the origin of the small value of ϵ'/ϵ in the Standard model. The phase of ϵ', $\delta_2 - \delta_0 + \pi/2 = (48 \pm 4)^0$ [3] is approximately equal to $\phi_K = (43.6 \pm 0.2)^0$, the phase of ϵ (see Sec. 4.1). Therefore ϵ'/ϵ is approximately real. Since this ratio is at most of order 10^{-3}, one expects the equality $\mathrm{Arg}\eta_{+-} = \mathrm{Arg}\eta_{00} = \phi_K$ to hold within a high precision if CPT invariance is valid.

A calculation of ϵ'/ϵ requires knowing the phases $\phi_0,\ \phi_2$. These can be estimated in the Standard Model using the tree and penguin diagrams as described in Fig. 3. Whereas the tree operator has real contributions to both A_0 and A_2, the penguin operator comes with a complex CKM phase and contributes only to A_0.

Figure 3: Tree and penguin diagrams in $K \to 2\pi$

Thus, one finds $\phi_2 = 0$ and ϕ_0 can be estimated from Fig. 3 to be given by

$$\tan\phi_0 \sim \frac{\mathrm{Im}(V_{td}V^*_{ts})}{V_{ud}V^*_{us}}(\frac{P}{T}) \sim \text{a few} \times 10^{-4}(\frac{P}{T}) \ . \tag{18}$$

With $P/T \sim \mathcal{O}(1)$ this implies $\epsilon'/\epsilon \sim \mathcal{O}(10^{-3})$. An actual calculation of ϵ'/ϵ is quite complicated [18], and involves large theoretical uncertainties in hadronic matrix elements of tree and penguin operators, on top of the experimental uncertainties in CKM elements. With the heavy t quark, additional electroweak penguin amplitudes (in which the gluon in Fig. 3 is replaced by a photon and by a Z boson) lead to complex contributions to A_2, through which ϕ_2 tends to cancel the ϕ_0 term in ϵ'. This enhances the uncertainty in ϵ'/ϵ. Any value in the range from a few times 10^{-5} to 10^{-3} seems to be possible. Measurement of a nonzero value for ϵ'/ϵ at a level of 10^{-4} would be an important observation by itself, however it cannot provide a precise test of the Standard Model.

5. THE B MESON SYSTEM

5.1 CP violation in $B^0 - \overline{B}^0$ Mixing

In the B system one has $|\Gamma_{12}| \ll |M_{12}|$. Γ_{12} is given by the absorptive part of the box diagram, Fig. 4(a), arising from decay channels which are common to B^0 and $\overline{B}^0$. On the other hand, M_{12} is the dispersive part of the diagram, Fig. 4(b), governed by the t quark mass. Crudely speaking $|\Gamma_{12}/M_{12}| \sim m_b^2/m_t^2$. Thus CP violation in $B^0 - \overline{B}^0$ mixing is expected to be very small in the Standard Model [19], $2\mathrm{Re}\epsilon_B \approx 1 - |q/p| \sim \mathcal{O}(10^{-3})$. This estimate, which is about the level of violation measured in the neutral K meso system, involves hadronic uncertainties and cannot provide a useful quantitative test of the Standard Model. A much larger value of $\mathrm{Re}\epsilon_B$ would be evidence against the CKM mechanism.

Figure 4: Box diagrams of Γ_{12} (a) and M_{12} (b)

CP violation in $B^0 - \overline{B}^0$ mixing is expected to show up as a charge asymmetry in semileptonic decays to "wrong charge" leptons, namely leptons to which only a mixed neutral B can decay:

$$A_{SL} = \frac{\Gamma(\overline{B}^0(t) \to \ell^+ \nu X) - \Gamma(B^0(t) \to \ell^- \overline{\nu} X)}{\Gamma(\overline{B}^0(t) \to \ell^+ \nu X) + \Gamma(B^0(t) \to \ell^- \overline{\nu} X)} \ . \tag{19}$$

$B^0(t)$ ($\overline{B}^0(t)$) is a time-evolving state, which is created as a B^0 ($\overline{B}^0$) state at $t = 0$. The asymmetry can be easily shown to be time-independent:

$$A_{SL} = \frac{1 - |q/p|^4}{1 + |q/p|^4} \approx 4\mathrm{Re}\epsilon_B \ . \tag{20}$$

There exists already an experimental upper limit from CLEO [20], $|\mathrm{Re}\epsilon_B| < 45 \times 10^{-3}$ (90% c.l.), which is about two orders of magnitude above the Standard Model prediction. It will be extremely difficult to observe an asymmetry at the level predicted by the model. However, further efforts to improve this limit, for the semileptonic decays which have a large branching ratio, is definitely worthwhile

Let us note in passing that while CP violation in the $B^0 - \overline{B}^0$ *mixing* i expected to be as small as about the one observed in $K^0 - \overline{K}^0$ mixing, the asymmetries expected in

neutral B *decays* are much larger than those of K decays. Thus, when discussing neutral B decay asymmetries in the following section we will take $|q/p| = 1$ which is a very good approximation. In this approximation the mixing parameter is a pure phase

$$\frac{q}{p} \approx \sqrt{\frac{M_{12}^*}{M_{12}}} \equiv e^{-2i\phi_M} = \begin{cases} e^{-2i\beta} & \text{for } B^0 \ , \\ 1 & \text{for } B_s^0 \ . \end{cases} \tag{21}$$

We will also assume $\Gamma_L = \Gamma_H$, which is a good approximation, in particular for B^0 where it is expected to hold with an accuracy better than 1%

5.2 CP violation in decays of mixed $B^0 - \overline{B}^0$

5.2.1 Time-dependent asymmetries in the general case

Consider the time-evolution of a state which is identified at time $t = 0$ as a B^0:

$$t = 0 : \qquad |B^0\rangle = \frac{e^{-i\phi_M}}{\sqrt{2}}(|B_L\rangle + |B_H\rangle) \ . \tag{22}$$

The time-evolutions of the states $B_{L,H}$ are given simply by their masses and by their equal decay width Γ: $|B_{L,H}(t=0)\rangle \to |B_{L,H}(t)\rangle = \exp[-i(m_{L,H} - \frac{i}{2}\Gamma)t]|B_{L,H}(t=0)\rangle$. Thus, the B^0 oscillates into a mixture of B^0 and $\overline{B}^0$:

$$t : \qquad |B^0(t)\rangle = e^{-i\overline{m}t}e^{-\frac{\Gamma}{2}t}[\cos(\frac{\Delta mt}{2})|B^0\rangle + ie^{-2i\phi_M}\sin(\frac{\Delta mt}{2})|\overline{B}^0\rangle] \ , \tag{23}$$

where $\overline{m} \equiv (m_H + m_L)/2$, $\Delta m \equiv m_H - m_L$. Now, assume that both B^0 and $\overline{B}^0$ can decay to a common state f, with amplitudes A and $\overline{A}$, respectively. The time-dependent decay rate to f of an initial B^0 and the corresponding rate for an initial $\overline{B}^0$ are

$$\Gamma(B^0(t) \to f) = e^{-\Gamma t}|A|^2[\cos^2(\frac{\Delta mt}{2}) + |\overline{A}/A|^2\sin^2(\frac{\Delta mt}{2}) - \mathrm{Im}(e^{-2i\phi_M}\overline{A}/A)\sin(\Delta mt)] \ ,$$

$$\Gamma(\overline{B}^0(t) \to f) = e^{-\Gamma t}|A|^2[|\overline{A}/A|^2\cos^2(\frac{\Delta mt}{2}) + \sin^2(\frac{\Delta mt}{2}) + \mathrm{Im}(e^{-2i\phi_M}\overline{A}/A)\sin(\Delta mt)] \ . \tag{24}$$

In the special case that f is an eigenstate of CP, $CP|f\rangle = \pm|f\rangle$, CP violation is manifest when $\Gamma(t) \equiv \Gamma(B^0(t) \to f) \neq \Gamma(\overline{B}^0(t) \to f) \equiv \overline{\Gamma}(t)$. The CP asymmetry is given by [21]:

$$Asym.(t) \equiv \frac{\Gamma(t) - \overline{\Gamma}(t)}{\Gamma(t) + \overline{\Gamma}(t)} = \frac{(1 - |\overline{A}/A|^2)\cos(\Delta mt) - 2\mathrm{Im}(e^{-2i\phi_M}\overline{A}/A)\sin(\Delta mt)}{1 + |\overline{A}/A|^2} \ . \tag{25}$$

The two terms in the numerator represent different sources of CP violation. The first term follows from CP violation in the direct decay of a neutral B meson, whereas the second term is induced by $B^0 - \overline{B}^0$ mixing.

5.2.2 Decay to CP eigenstates dominated by a single CKM phase

Let us first consider the case of no direct CP violation, $|\overline{A}| = |A|$, in whic a single weak amplitude (or rather a single weak phase) dominates the decay [22]. This is the case of a maximal interference term in Eq.(24). Denoting the weak and strong phases by ϕ_f and δ, respectively, we have $A = |A|\exp(i\phi_f)\exp(i\delta)$, $\overline{A} = \pm|\overline{A}|\exp(-i\phi_f)\exp(i\delta)$, and the asymmetry is given simply by

$$Asym.(t) = \pm\sin 2(\phi_M + \phi_f)\sin(\Delta m t)~. \tag{26}$$

The sign is given by $CP(f)$. The time-integrated asymmetry is

$$Asym. = \pm(\frac{\Delta m/\Gamma}{1 + (\Delta m/\Gamma)^2})\sin 2(\phi_M + \phi_f)~. \tag{27}$$

That is, in this case *the CP asymmetry measures a CKM phase with no hadronic uncertainty.* The integrated asymmetry in B^0 decays may be as large as $(\Delta m/\Gamma)/[1 + (\Delta m/\Gamma)^2] = 0.47$.

The best example is the well-known and much studied case [23] of $B^0 \to \psi K_S$, for which a branching ratio of about 5×10^{-4} has already been measured [24]. In this case $\phi_M = \beta,~\phi_f = \mathrm{Arg}(V^*_{cb}V_{cs}) = 0,~CP(\psi K_S) = -1$. Another case is $B^0 \to \pi^+\pi^-$, for which a combined branching ratio $BR(B^0 \to \pi^+\pi^-$ and $K^+\pi^-) = (2.3 \pm 0.8) \times 10^{-5}$ has been measured [25], with a likely solution in which the two modes have about equal branching ratios. In this case $\phi_f = \mathrm{Arg}(V^*_{ub}V_{ud}) = \gamma,~CP(\pi^+\pi^-) = 1$. Consequently one has in these two cases

$$Asym.(B^0 \to \psi K_S; t) = -\sin 2\beta\sin(\Delta m t)~,$$

$$Asym.(B^0 \to \pi^+\pi^-; t) = -\sin 2\alpha\sin(\Delta m t)~. \tag{28}$$

In the case of decay to two pions the asymmetry obtains, however, corrections from a second (penguin) CKM phase. This problem will be discussed below.

5.2.3 Decay to non-CP eigenstates

Angles of the unitarity triangle can also be determined from neutral B decays to states f which are not eigenstates of CP [26]. This is feasible when both a B^0 and a $\overline{B}^0$ can decay to a final state which appears in only one partial wave, provided that a single CKM phase dominates each of the corresponding decay amplitudes.

The time-dependent rates for states which are B^0 or $\overline{B}^0$ at $t = 0$ and decay at time t to a state f or its charge-conjugate $\overline{f}$ are given by [27]:

$$\Gamma_f(t) = e^{-\Gamma t}[|A|^2\cos^2(\frac{\Delta m t}{2}) + |\overline{A}|^2\sin^2(\frac{\Delta m t}{2}) + |A\overline{A}|\sin(\Delta\delta + \Delta\phi_f + 2\phi_M)\sin(\Delta m t)]~,$$

$$\overline{\Gamma}_f(t) = e^{-\Gamma t}[|\overline{A}|^2\cos^2(\frac{\Delta mt}{2}) + |A|^2\sin^2(\frac{\Delta mt}{2}) - |A\overline{A}|\sin(\Delta\delta + \Delta\phi_f + 2\phi_M)\sin(\Delta mt)] \ ,$$

$$\Gamma_{\overline{f}}(t) = e^{-\Gamma t}[|\overline{A}|^2\cos^2(\frac{\Delta mt}{2}) + |A|^2\sin^2(\frac{\Delta mt}{2}) - |A\overline{A}|\sin(\Delta\delta - \Delta\phi_f - 2\phi_M)\sin(\Delta mt)] \ ,$$

$$\overline{\Gamma}_{\overline{f}}(t) = e^{-\Gamma t}[|A|^2\cos^2(\frac{\Delta mt}{2}) + |\overline{A}|^2\sin(\frac{\Delta mt}{2}) + |A\overline{A}|\sin(\Delta\delta - \Delta\phi_f - 2\phi_M)\sin(\Delta mt)] \ . \quad (29)$$

Here $\Delta\delta$, $(\Delta\phi_f)$ is the difference between the strong (weak) phases of A and $\overline{A}$. The four rates depend on four unknown quantities, $|A|$, $|\overline{A}|$, $\sin(\Delta\delta + \Delta\phi_f + 2\phi_M)$, $\sin(\Delta\delta - \Delta\phi_f - 2\phi_M)$. Measurement of the rates allows a determination of the weak CKM phase $\Delta\phi_f + 2\phi_M$ apart from a two-fold ambiguity [26].

There are two interesting examples to which this method can be applied. In the first case, $B^0 \to \rho^+\pi^-$, one must neglect a second contribution of a penguin amplitude, a problem which will be addressed in the following subsection. Assuming for a moment that tree diagrams, shown in Figs. 5(a), 5(b), dominate A and $\overline{A}$, one can measure in this manner the angle α, since in this case $\Delta\phi_f + 2\phi_M = 2(\gamma + \beta) = 2(\pi - \alpha)$. A second case, which may be used to measure γ, is $B_s^0 \to D_s^+K^-$, in which only one amplitude contributes to A and another amplitude - to $\overline{A}$.

Figure 5: Diagrams of $B^0 \to \rho^+\pi^-$ (a) and $\overline{B}^0 \to \rho^+\pi^-$ (b)

5.2.4 Corrections from penguin amplitudes

A crucial question is, of course, how good is the assumption of a single dominant CKM phase, which is needed for a clean determination of an angle of the unitarity triangle. One may try to answer this question experimentally by looking for an extra $\cos(\Delta m t)$ term in the time-dependent asymmetry o Eq.(25) which describes CP violation in the direct decay of B^0. There is, however, the danger that this term will be unobservably small, just because the final state interaction phase difference happens to be small. The effect of second amplitude on the coefficient of $\sin(\Delta m t)$, which is proportional to the cosine of this phase-difference [21], may still be large. This will be demonstrated below for $B^0 \to \pi^+\pi^-$.

In a wide variety of decay processes there exists a second amplitude due to "penguin" diagrams [28] in addition to the usual "tree" diagram. In general, the new contribution becomes more disturbing when the process involves a stronger CKM-suppression. The penguin-to-tree ratio of amplitudes is proportinal to the ratio of the corresponding CKM factors and to a QCD fator $(\alpha_s(m_b^2)/12\pi)\ln(m_t^2/m_b^2)$. This ratio may be estimated for a given process. A few examples of final states in B^0 decays, with different levels of CKM suppression, are [21]:

$$\frac{\text{Penguin}}{\text{Tree}} = \begin{cases} 10^{-3} & \psi K_S \ , \\ 0.05 & D^+D^- \ (D^{*+}D^-) \ , \\ 0.20 & \pi^+\pi^- \ (\rho^+\pi^-) \ , \\ \mathcal{O}(1) & K_S\pi^0 \ . \end{cases} \tag{30}$$

These numbers represent quite crude estimates, since there exists no reliable method to calculate hadronic matrix elements of penguin operators. One way to obtain information about these matrix elements would be to measure pure penguin processes, such as $B^0 \to \phi K_S$. Another way will be mentioned when discussing charged B decays.

We see from Eqs.(30) that the decay $B^0 \to \psi K_S$ remains a pure case, with less than 1% corrections, also in the presence of penguin contributions. On the other hand, penguin effects on the CP asymmetry of $B^0 \to \pi^+\pi^-$ may be substantial. This is demonstrated in Fig. 6, taken from Ref. 29, which shows the coefficient of the $\sin(\Delta m t)$ term in the asymmetry as function of the angle α for a zero final state interaction phase difference. The range of values comes from taking the ratio (Penguin/Tree) to be anywhere between 0.04 and 0.20. For a ratio of 0.20, an asymmetry as large as 0.40 can possibly be measured even when $\sin(2\alpha) = 0$.

Figure 6: Asymmetry in $B^0 \to \pi^+\pi^-$ as function of α

5.2.5 Removing penguin corrections in $B^0 \to \pi^+\pi^-$

It is possible to disentangle the penguin contribution in $B^0 \to \pi^+\pi^-$ fro the tree-dominating asymmetry by measuring also the rates of $B^+ \to \pi^+\pi^0$ and $B^0 \to \pi^0\pi^0$. The method [30] is based on the observation that the two weak operators contributing to the three isospin-related processes have different isospin properties just as in $K \to 2\pi$. Whereas the tree operator is a mixture of $\Delta I = 1/2$ and $\Delta I = 3/2$, the penguin operator is pure $\Delta I = 1/2$. Denoting the physical amplitudes of $B \to \pi^+\pi^-, \pi^0\pi^0, \pi^+\pi^0$ by the charges of the two corresponding pions, one finds from an isospin decomposition

$$\frac{1}{\sqrt{2}} A^{+-} = A_2 - A_0 \ , \quad A^{00} = 2A_2 + A_0 \ , \quad A^{+0} = 3A_2 \ , \tag{31}$$

where A_0 and A_2 are the amplitudes for a B^0 or a B^+ to decay into a $\pi\pi$ state with $I = 0$ and $I = 2$, respectively. This yields the complex triangle relation

$$\frac{1}{\sqrt{2}} A^{+-} + A^{00} = A^{+0} \ . \tag{32}$$

There is a similar triangle relation for the charge-conjugated processes:

$$\frac{1}{\sqrt{2}} \overline{A}^{+-} + \overline{A}^{00} = \overline{A}^{-0} \ . \tag{33}$$

Here, $\overline{A}^{+-}$, $\overline{A}^{00}$, and $\overline{A}^{-0}$ are the amplitudes for the processes $\overline{B}^0 \to \pi^+\pi^-$, $\overline{B}^0 \to \pi^0\pi^0$, and $B^- \to \pi^-\pi^0$, respectively. The $\overline{A}$ amplitudes are obtained from the A amplitudes by simply changing the sign of the CKM phases (the strong phases remain the same).

The crucial point in the analysis is that the "tree" contribution to A_2 has a well-defined weak phase, which is given by the angle γ of the unitarity triangle. (The electroweak penguin contribution to A_2 is negligible).

$$A_2 = |A_2|e^{i\delta_2}e^{i\gamma} \ , \quad \overline{A}_2 = |A_2|e^{i\delta_2}e^{-i\gamma} \ . \tag{34}$$

where δ_2 is the $I = 2$ final-state-interaction phase. It is convenient to define $\tilde{A} = \exp(2i\gamma)\overline{A}$ so that $\tilde{A}_2 = A_2$ and $\tilde{A}^{-0} = A^{+0}$. The two complex triangles representing Eqs. (32)(33) (where $\overline{A}$ is replaced by $\tilde{A}$) are shown in Fig. 7. They have a common base (CP is conserved in $B^+ \to \pi^+\pi^0$); however the length of their corresponding sides are different. That is, CP is violated in $B^0 \to \pi^+\pi^-$ and in $B^0 \to \pi^0\pi^0$.

Figure 7: Isospin triangles of $B \to \pi\pi$

The six sides of the two triangles are measured by the decay rates of $B^\pm$ and by the time-integrated rates of B^0 ($\overline{B}^0$). This determines the two triangles within a two-fold ambiguity; each triangle may be turned up-side-down. The coefficients of the $\sin(\Delta m t)$ term in $B^0 \to \pi^+\pi^-$ measures the quantities

$$\mathrm{Im}\,(e^{-2i(\beta+\gamma)}\frac{\tilde{A}^{+-}}{A^{+-}}) = \frac{|\tilde{A}^{+-}|}{|A^{+-}|}\sin(2\alpha + \theta_{+-}), \tag{35}$$

where θ_{+-} is obtained from Fig. 7. (This angle vanishes in the absence of the penguin correction). This determines the angle α.

The application of this method in asymmetric e^+e^- B-factories [31] is likely to suffer from a very small branching ratio of $B^0 \to \pi^0\pi^0$ (which is expected to be color-suppressed) and from the difficulty of observing two neutral pions. Of course, if the penguin term is

small, its effect on the asymmetry of $B^0 \to \pi^+\pi^-$ will be small. When discussing charged B decays we will mention how these decays can tell us something about the magnitude of the penguin term in $B^0 \to \pi\pi$.

A similar isospin analysis was carried out for other decays in which penguin amplitudes are involved [32]. In general, the precision of determining a CKM phase becomes worse when a larger number of amplitudes must be related. Also a few ambiguities show up in this case. In the case of $B^0 \to \rho\pi$ (and $B^+ \to \rho\pi$) five physical decay amplitudes appear. In this case the ambiguity can be resolved if a full Dalitz plot analysis can be made for the three pion final states [33].

5.3. CP Violation in charged B decays

5.3.1 A theoretical difficulty

The simplest manifestations of CP violation are different partial decay widths for a particle and its antiparticle into corresponding decay modes. Consider a general decay $B^+ \to f$ and its charge-conjugate process $B^- \to \overline{f}$. In order that these two proceses have different rates, two amplitudes (A_1, A_2) must contribute, with different CKM phases $(\phi_1 \neq \phi_2)$ and different final state interaction phases $(\delta_1 \neq \delta_2)$:

$$A(B^+ \to f) \ = \ |A_1|e^{i\phi_1}e^{i\delta_1} \ + \ |A_2|e^{i\phi_2}e^{i\delta_2} \ ,$$

$$\overline{A}(B^- \to \overline{f}) \ = \ |A_1|e^{-i\phi_1}e^{i\delta_1} \ + \ |A_2|e^{-i\phi_2}e^{i\delta_2} \ ,$$

$$|A|^2 - |\overline{A}|^2 = 2|A_1A_2|\sin(\phi_1 - \phi_2)\sin(\delta_1 - \delta_2) \ . \tag{36}$$

The theoretical difficulty of relating an asymmetry in charged B decays to a pure CKM phase follows from having two unknowns in the problem: The ratio of amplitudes, $|A_2/A_1|$, and the final state phase difference, $\delta_2 - \delta_1$. Both quantities involve quite large theorertical uncertainties.

This is demonstrated in Fig. 8, which describes the two amplitudes A_1 and A_2 for $B^+ \to K^+\pi^0$, given by the “penguin” and “tree” diagrams, respectively.

Figure 8: Penguin (a) and tree (b) diagrams in $B^+ \to K^+\pi^0$

In this case $\phi_1 = \pi$, $\phi_2 = \gamma$. A few calculations of the asymmetry in this process exist [34], based on model-dependent estimates of the tree-to-penguin ratio of amplitudes and of the strong phase difference. The strong phase includes a phase due to the absorptive part of the physical $c\bar{c}$ quark pair in the penguin diagram, which may be viewed as describing rescattering processes such as $B \to \overline{D}D_s \to K\pi$. All such model-dependent calculations involve large theoretical uncertainties.

5.3.2 Measuring γ in $B^\pm \to D^0K^\pm$

The decays $B^\pm \to D^0_1(D^0_2)K^\pm$ and a few other processes of this type provide a unique case [35], in which one can measure separately the magnitudes of the two contributing amplitudes, and thereby determine the CKM phase γ. $D^0_1(D^0_2) = (D^0 + (-)\overline{D}^0)/\sqrt{2}$ is a CP-even (odd) state, which is identified by its CP-even (odd) decay products. For instance, the states $K_S\pi^0$, $K_S\rho^0$, $K_S\omega$, $K_S\phi$ identify a D^0_2, while $\pi^+\pi^-$, K^+K^- represent a D^0_1. The decay amplitudes of the above two charge-conjugate processes can be written (say for D^0_1) in the form

$$\begin{aligned} \sqrt{2}A(B^+ \to D^0_1K^+) &= |A_1|\exp(i\gamma)\exp(i\delta_1) + |A_2|\exp(i\delta_2)\ , \\ \sqrt{2}A(B^- \to D^0_1K^-) &= |A_1|\exp(-i\gamma)\exp(i\delta_1) + |A_2|\exp(i\delta_2). \end{aligned} \tag{37}$$

A_1 and A_2 are the two weak amplitudes, shown in Fig. 9(b) and 9(a), respectively. Their CKM factors $V^*_{ub}V_{cs}$ and $V^*_{cb}V_{us}$ are of comparable magnitudes. Their weak phases are γ and zero. Since A_1 leads to final states with isospin 0 and 1, whereas A_2 can only lead to isospin 1 states, one generally expects [36] $\delta_1 \neq \delta_2$.

Figure 9: Diagrams decribing $B^+ \to \overline{D}^0K^+$ (a) and $B^+ \to D^0K^+$ (b)

As shown in Fig. 9, the two amplitudes on the right-hand-sides of the first of Eqs. (37)

are the amplitudes of $B^+ \to D^0K^+$ and $B^+ \to \overline{D}^0 K^+$, respectively. Similarly, the two terms in the second equation describe the amplitudes of $B^- \to \overline{D}^0 K^-$ and $B^- \to D^0K^-$, respectively. The flavor states D^0 and $\overline{D}^0$ are identified by the charge of the decay lepton or kaon. Thus one has:

$$\begin{aligned} \sqrt{2}A(B^+ \to D_1^0K^+) &= A(B^+ \to D^0K^+) \ + \ A(B^+ \to \overline{D}^0 K^+), \\ \sqrt{2}A(B^- \to D_1^0K^-) &= A(B^- \to \overline{D}^0 K^-) \ + \ A(B^- \to D^0K^-). \end{aligned} \tag{38}$$

Eqs. (38) can be described by two triangles in the complex plane as shown in Fig. 10.

Figure 10: Triangles describing Eqs.(38)

The two triangles represent the complex B^+ and B^- decay amplitudes. Note that

$$\begin{aligned} A(B^+ \to \overline{D}^0 K^+) = A(B^- \to D^0K^-) \qquad &, \\ A(B^+ \to D^0K^+) = \exp(2i\gamma)A(B^- \to \overline{D}^0 K^-)&, \\ |A(B^+ \to D_1^0K^+)| \neq |A(B^- \to D_1^0K^-)| \qquad &. \end{aligned} \tag{39}$$

This implies that CP is conserved in $B^\pm \to D^0(\overline{D}^0)K^\pm$ but is violated in $B^\pm \to D_1^0K^\pm$. In the last of Eqs.(39) we assumed $\gamma \neq 0$, $\delta_1 \neq \delta_2$. The asymmetry in the rates of $B^\pm \to D_1^0K^\pm$ depends on γ and $\delta_2 - \delta_1$; clearly

$$\begin{aligned} &|A(B^+ \to D_1^0K^+)|^2 - |A(B^- \to D_1^0K^-)|^2 \\ &= 2|A(B^+ \to \overline{D}^0 K^+)||A(B^+ \to D^0K^+)| \sin(\delta_2 - \delta_1) \sin\gamma. \end{aligned} \tag{40}$$

The procedure for obtaining γ is straightforward. Measurements of the rates of the above six proccesses, two pairs of which are equal, determine the lengths of all six sides of the two triangles. When the two triangles are formed, 2γ is the angle between $A(B^+ \to D^0K^+)$ and $A(B^- \to \overline{D}^0 K^-)$. This determines the magnitude of γ within a two-fold ambiguity related to a possible interchange of γ and $\delta_1 - \delta_2$. This ambiguity may be resolved by carrying out

this analysis for other decay processes of the type $B^\pm \to D^0(\overline{D}^0, D^0_{1(2)})X^\pm$, where $X^\pm$ is any other state with the flavor quantum number of a $K^\pm$.

The feasibility of observing a CP asymmetry in $B^+ \to D^0_{1(2)}K^+$ depends on the branching ratios of the three related decay processes, and on the values of the weak and strong phases. One may estimate $BR(B^+ \to \overline{D}^0 K^+) \approx 2 \times 10^{-4}$, using the corresponding measured Cabibbo-allowed branching ratio of $B^+ \to \overline{D}^0\pi^+$ [24]. The process $B^+ \to D^0K^+$, in which the two quarks of the $c\overline{s}$ current enter two different meson states, is likely to be "color-suppressed". Color suppression has already been seen in $B \to D\pi$ [24]. If the same suppression factor applies also to $B^+ \to D^0K^+$, then the branching ratio of this process is at most at the level of 10^{-5}. Using a value of 5×10^{-6}, the feasibility for observing a CP asymmetry in $B^+ \to D^0_{1(2)}K^+$ was studied [37] as function of γ and $\delta_2 - \delta_1$, for a (symmetric) $e^+e^- \to \Upsilon(4S)$ B-factory with an integrated luminosity of $20 fb^{-1}$. The discovery region was found to cover a significant part of the $(\gamma, \delta_2 - \delta_1)$ plane. For small final state phase differences the experiment is sensitive mainly to values of γ around 90^0. Large values of $\delta_2 - \delta_1$ allow a useful measurement of γ in the range $50^0 \le \gamma \le 130^0$.

Present experiments are reaching the level of being able to observe the first Cabibbo suppressed decays $B \to DK$. The question of color-suppression in these decays needs to be studied. It is possible that the final state phase difference $\delta_2 - \delta_1$ is too small to allow a good measurement of γ if this angle is not around 90^0. A recent study [38] generalized this method to quasi-two-body decays $B \to DK_i \to DK\pi$, where K_i are excited kaon resonance states with masses around 1400 MeV. The resonance effect gives give rise to large final state phases and thus enhances the CP asymmetry.

5.3.3 Using SU(3) to determine γ from $B^+ \to \pi K$ and $B^+ \to \pi\pi$

Flavor SU(3) symmetry can be used to relate B decays to $\pi\pi$, πK and KK states [39]. Recently this idea was applied [40] jointly with the dynamical assumption that annihilation-like diagrams are small in these two-body decays. This assumption means that certain rescattering effects are small. That is, final states which are produced, for instance, by tree decay amplitudes have small rescattering amplitudes to states created by quark-antiquark annihilation. This assumption is motivated by the high B meson mass (compared to f_B). It is supported by the experimental evidence for color-suppression and for factorization in two body B decays [24], two features which are expected to be spoiled by large rescattering amplitudes. It was shown that the assumption of negligible rescattering is equivalent to assuming that certain final state phases are equal to others [41]. Neglecting such rescattering effects leads to simple testable predictions, such as $A(B^0 \to K^+K^-) = 0$, and to useful

information about weak and strong phases. Here we wish to demonstrate this idea through a rather simple case [42].

Consider the decay $B^+ \to \pi^0 K^+$ for which the two contributing amplitudes A_1 and A_2 are described in Fig. 8. The penguin amplitude A_1 is related by isospin to the amplitude of $B^+ \to \pi^+ K^0$, in which the annihilation contribution is neglected, $A_1 = A(\pi^+ K^0)/\sqrt{2}$. The tree amplitude A_2 is related by SU(3) to the amplitude of $B^+ \to \pi^+\pi^0$, which receives no penguin contribution. Using factorization to introduce SU(3) breaking into this relation, one has $A_2 = (f_K/f_\pi)|V_{us}/V_{ud}|A(\pi^+\pi^0)$. Thus one obtains a simple relation between the three B^+ decay amplitudes:

$$A(\pi^0 K^+) = \frac{1}{\sqrt{2}} A(\pi^+ K^0) + \frac{f_K}{f_\pi} |\frac{V_{us}}{V_{ud}}| A(\pi^+\pi^0) \ . \tag{41}$$

A similar relation holds among the corresponding B^- decay amplitudes. There are phase relations, $A(\pi^- \overline{K}^0) = A(\pi^+ K^0), A(\pi^-\pi^0) = \exp(-2i\gamma)A(\pi^+\pi^0)$, since the weak phases of these amplitudes are π and $-\gamma$, respectively. The two relations among the B^+ and among the B^- amplitudes are analogous to Eqs.(38). They can be descrlibed by two triangles very similar to those of Fig. 10. In the present case the two triangles share a common base given by $A(\pi^- \overline{K}^0) = A(\pi^+ K^0)$, and the angle between the sides describing $A(\pi^-\pi^0)$ and $A(\pi^+\pi^0)$ is 2γ. Measurements of the four rates into $\pi^0 K^+, \pi^0 K^-, \pi^+ K^0, \pi^+\pi^0$, suffices to determine γ. CP violation is demonstrated by $A(\pi^0 K^+) \neq A(\pi^0 K^-)$. About 100 events of this mode, which is expected to have a branching ratio of about 10^{-5}, are needed to measure γ to a statistical accuracy of 10^0 [42].

This method is not as clean as the one using $B^\pm \to D^0 K^\pm$ decays, since it is based on certain dynamical assumptions. Also, it was recently noted [43] that contributions from electroweak penguin diagrams can spoil the relation (41). Information about the effect of these diagrams, of SU(3) breaking and of annihilation-like diagrams, and the separate magnitudes of tree and penguin amplitudes, can be obtained from a systematic study of all the possible B decay modes to two light pseudoscalar mesons [40]. Such a detailed study may then be used to evaluate the precision to which the weak phase can be determined.

5.4 Flavor-tagging of neutral B mesons

5.4.1 Tagging by the associated B decay

In order to measure CP asymmetries in neutral B decays one must identify the flavor of the decaying meson at some reference time $t = 0$. In a $e^+e^- \to \Upsilon(4S)$ B-factory this is achieved [23] by observing a lepton, or a cascade charged kaon from $B \to D \to K$, from the

decay of the othe neutral B. Since at any time after production the two neutral B mesons form *coherent* $C(B^0\overline{B}^0) = -1$ EPR pair, the charge of the lepton serves to "tag" the opposite flavor of the other B at the time of semileptonic decay. Furthermore, the CP asymmetry is odd in the time-difference of the two decays, and consequently asymmetric storage rings are required for an asymmetry measurement.

A similar method of determining the flavor of neutral B mesons in high energy e^+e^- or in hadronic collisions [44] uses as a "tag" the lepton from a semileptonic decay of an associated b-meson or b-baryon. The flavor is misidentified part of the time as a result of $B^0 - \overline{B}^0$ mixing. The probability of misidentification and its effect on diluting the measured CP asymmetry can be crudely estimated. Since the B^0 and $\overline{B}^0$ are usually produced with many other particles, it is commonly assumed that the are in an *incoherent* mixture.

5.4.2 Tagging by correlated charged pions

An alternative method of flavor identification [45] uses an expected correlation between the decaying neutral B and a charged pion making a low-mass $B - \pi$ system. There are two arguments for such a correlation. The first argument is based on the existence of positive-parity "B^{**}" resonances, with $J^P = 0^+,\ 1^+,\ 2^+$ and masses below about 5.8 GeV/c^2 [46]. Using Heavy Quark Symmetry, this mass value is obtained from the corresponding observed "D^{**}" masses (2420, 2460 GeV/c^2). The B^{**} resonances decay to $B\pi$ and/or $B^*\pi$ mesons in $I = 1/2$ states. That is, a π^+ will accompany a B^0 and not a $\overline{B}^0$. A similar method [47] has been used to tag neutral charmed mesons, where the decays $D^{*+} \to D^0\pi^+,\ D^{*-} \to \overline{D}^0\pi^-$ are kinematically allowed. The second argument is that in b-quark fragmentation the leading pion carries information about the flavor of the neutral B. A neutral B meson containing an initially produced b quark is a $\overline{B}^0$ which contains a $\overline{d}$ quark. The next charged pion down the fragmentation chain must contain a d, and hence must be a π^-. Similarly, a B^0 will be correlated with a π^+.

The efficiency of this method depends on the degree of the correlation, which can be studied in neutral B decays to states of identified flavor, such as $D^-\pi^+$ or ψK^{*0} (with $K^{*0} \to K^+\pi^-$). Usually B mesons are produced in an isospin-independent manner and one can find this correlation using charged B mesons as well. The time-dependent CP asymmetry measured with this tagging method is diluted by the degree of correlation. Aside from using the asymmetry to determine weak phases, it can also be used to test the assumption that the produced B^0 and $\overline{B}^0$ are incoherent with respect to one another [48].

6. SUMMARY

The observed CP violation in $K^0\overline{K}^0$ mixing is successfully parametrized in terms of a phase in the CKM matrix. This phase is largely unknown at present. Tests of the Standard Model of CP violation require more precise information about magnitudes and phases of CKM elements. Future K decay experiments may have the potential of measuring a nonzero value for ϵ'/ϵ, thus confirming the expected phenomenon of direct CP violation in K decays. However, due to theoretical uncertainties this cannot provide a precise test of the Standard Model and cannot cleanly determine CKM parameters. On the other hand, measurements of certain CP asymmetries in B decays can determine CKM phases in manners which are free of hadronic uncertainties. At the very least, this will allow direct measurements of these fundamental parameters. With an improved knowledge of the magnitudes of CK elements, this may eventually serve to overconstrain the CKM matrix. One would hope to find some inconsistencies which could be clues for physics beyond the Standard Model. Afterall, the observed baryon asymmetry in the universe seems to require other sources of CP violation [49].

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REFERENCES

1. J. H. Christenson, J. W. Cronin, V. L. Fitch and R. Turlay, *Phys. Rev. Lett.* **13** (1964) 138.
2. J. W. Cronin, *Rev. Mod. Phys.* **53** (1981) 373; K. Kleinknecht, in *CP Violation*, ed. C. Jarlskog (Singapore, World Scientific, 1989), p. 41.
3. Particle Data Group, *Review of Particle Properties, Phys. Rev.* **D50** (1994) 1173.
4. E731 Collaboration, L. K. Gibbons *et al.*, *Phys. Rev. Lett.* **70** (1993) 1203.
5. NA31 Collaboration, G. D. Barr *et al.*, *Phys. Lett.* **B317** (1993) 233.
6. N. W. Tanner and R. H. Dalitz, *Ann. Phys. (N.Y.)* **171** (1986) 463; T. Nakada, in *Proceedings of the XVI International Symposium on Lepton and Photon Interactions*, Cornell University, August 10-15, 1993, ed. P. Drell and D. Rubin (New York, AIP,

1994), p. 425.

7. Particle Data Group, *Review of Particle Properties, Phys. Rev.* **D45** (1992) 1.
8. E731 Collaboration, L. K. Gibbons *et al.*, *Phys. Rev. Lett.* **70** (1993) 1199; E773 Collaboration, EFI 94-31, presented by B. Schwingenheuer at the Fifth Conference on the Intersections of Particle and Nuclear Physics, St. Petersburg, Florida, June 1994; CPLEAR Collaboration, C. Yeche *et al.*, DAPNIA-SPP-94-18, June 1994.
9. B. Winstein and L. Wolfenstein, *Rev. Mod. Phys.* **65** (1993) 1113.
10. S. L. Glashow, *Nucl. Phys.* **22** (1961) 579; S. Weinberg, *Phys. Rev. Lett.* **19** (1967) 1264; A. Salam, in *Elementary Particle Theory*, ed. N. Svartholm (Almqvist and Wiksell, Stockholm, 1968).
11. N. Cabibbo, *Phys. Rev. Lett.* **10** (1963) 531; M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49** (1973) 652.
12. *CP Violation*, *op. cit.*; Y. Nir and H. Quinn, *Ann. Rev. Nucl. Part. Sci.* **42** (1992) 211; B. Winstein and L. Wolfenstein, Ref.9; J. L. Rosner, Enrico Fermi Institute Report EFI 94-25, presented at PASCOS 94 Conference, Syracuse, NY, May 1994.
13. S. Stone, Syracuse University Report HEPSY-94-5, presented at PASCHOS 94 Conference, *op. cit.*; A. Ali and D. London, CERN Report CERN-TH.7398/94, August 1994.
14. L. Wolfenstein, *Phys. Rev. Lett.* **51** (1983) 1945.
15. A. Ali and D. London, Ref.13.
16. R. Aleksan, B. Kayser and D. London, *Phys. Rev. Lett.* **73** (1994) 18.
17. T. Inami and C. S. Lim, *Prog. Theor. Phys.* **65** (1981) 297; A. J. Buras, W. Slominski and H. Steger, *Nucl. Phys.* **B238** (1984) 529; **B245** (1984) 369.
18. G. Buchalla, A. J. Buras and M. K. Harlander, *Nucl. Phys.* **B337** (1990) 313; J. Heinrich, E.A. Paschos, J. M. Schwarz and Y. L. Wu, *Phys. Lett.* **B279** (1992) 140; A. J. Buras, M. Jamin and M. E. Lautenbacher, *Nucl. Phys.* **408** (1993) 209; R. Ciuchini, E. Franco, G. Martinelli and L. Reina, *Phys. Lett.* **B301** (1993) 263.
19. I. I. Bigi, V. A. Khoze, N. G. Uraltsev and A. I. Sanda, in *CP Violation*, *op. cit.*, p. 175.
20. CLEO Collaboration, J. Bartelt *et al.*, *Phys. Rev. Lett.* **71** (1993) 1680.
21. M. Gronau, *Phys. Rev. Lett.* **63** (1989) 1451.
22. A.B. Carter and A.I. Sanda, *Phys. Rev. Lett.* **45** (1980) 952; *Phys. Rev.* **D23** (1981) 1567; I.I. Bigi and A.I. Sanda *Nucl. Phys.* **B193** (1981) 85; **B281** (1987) 41; I. Dunietz and J.L Rosner, *Phys. Rev.* **D34** (1986) 1404.
23. *The Physics Program of a High-Luminosity Asymmetric B-Factory at SLAC*, ed. D.

Hitlin, SLAC-PUB-353 (1989) and SLAC-PUB-373 (1991); *Physics Rationale for a B-Factory*, K. Lingel *e al.*, CLNS 91-1043 (1991); *Physics and Detector at KEK Asymmetric B Factory*, K. Abe *et al.*, KEK Report 92-3 (1992).

24. CLEO Collaboration, M. S. Alam *et al.*, *Phys. Rev.* **D50** (1994) 43.
25. CLEO Collaboration, M. Battle *et al.*, *Phys. Rev. Lett.* **71** (1993) 3922.
26. R. Aleksan, I. Dunietz, B. Kayser and F. Le Diberder, *Nucl. Phys.* **B361** (1991) 141; R. Aleksan, I. Dunietz and B. Kayser, *Zeit. Phys.* **C54** (1992) 653.
27. M. Gronau, *Phys. Lett.* **B233** (1989) 479.
28. M. Gronau, Ref.21; D. London and R.D. Peccei, *Phys. Lett.* **B223** (1989) 257; B. Grinstein, *Phys. Lett.* **B229** (1989) 280.
29. M. Gronau, *Phys. Lett.* **B300** (1993) 163.
30. M. Gronau and D. London, *Phys. Rev. Lett.* **65** (1990) 3381.
31. R. Aleksan, A. Gaidot and G. Vasseur, in *B Factories, The Stat of the Art in Accelerators, Detectors and Physics*, ed. D. Hitlin, SLAC-400 (1992), p. 588.
32. Y. Nir and H. Quinn, *Phys. Rev. Lett.* **67** (1991) 541; H. J. Lipkin, Y. Nir, H. R. Quinn and A. E. Snyder, *Phys. Rev.* **D44** (1991) 1454; M. Gronau, *Phys. Lett.* **B265** (1991) 389; L. Lavoura, ıMod. Phys. Lett. **A7** (1992) 1553.
33. H. R. Quinn and A. E. Snyder, *Phys. Rev.* **D48** (1993) 2139.
34. L. L. Chau and H. Y. Cheng, *Phys. Rev. Lett.* **59** (1987) 958; N. Deshpande and J. Trampetic, *Phys. Rev.* **D41** (1990) 2926; J. M. Gerard and W. S. Hou, *Phys. Rev.* **D43** (1991) 2909; H. Simma, G. Eilam and D. Wyler, *Nucl. Phys.* **B352** (1991) 367.
35. M. Gronau and D. Wyler, *Phys. Lett.* **B265** (1991) 172. See also M. Gronau and D. London, *Phys. Lett.* **B253** (1991) 483; I. Dunietz, P̑hys. Lett **B270** (1991) 75.
36. I. I. Bigi and A. I. Sanda, *Phys. Lett.* **B211** (1988) 213.
37. S. Stone, in *Beauty 93, Proceedings of the First International Workshop on B Physics at Hadron Machines*, Liblice Castle, Melnik, Czech Republic, Jan. 18-22, 1993; ed. P. E. Schlein, *Nucl. Instrum. Meth.* **333** (1993) 15.
38. D. Atwood, G. Eilam, M. Gronau and A. Soni, CERN Report CERN-TH.7428/94, September 1994.
39. D. Zeppenfeld, *Zeit. Phys.* **C8** (1981) 77; M. Savage and M. Wise, *Phys. Rev.* **D39** (1989) 3346; *ibid.* **40** (1989) 3127(E); J. Silva and L. Wolfenstein, *Phys. Rev.* **D49** (1994) R1151.
40. M. Gronau, O. F. Hernández, D. London, and J. L. Rosner, *Phys. Rev.* **D50** (1994) Oct. 1; *Phys. Lett.* **B333** (1994) 500; see also A. J. Buras and R. Fleischer, Max-Planck-Institute Report MPI-PhT/94-56 August 1994.

41. See also L. Wolfenstein, to be published in *Phys. Rev.* **D50** (1994).
42. M. Gronau, J. L. Rosner and D. London, *Phys. Rev. Lett.* **73** (1994) 21.
43. N. G. Deshpande and X-G He, University of Oregon Report OITS-553, August 1994.
44. See e.g. *B Physics at Hadron Accelerators, Proceedings of a workshop at Fermilab*, Nov., 1992, ed. J.A. Appel (Fermilab, Batavia, Il, 1992); *Beauty 93*, *op. cit.*; M. Chaichian and A. Fridman, *Phys. Lett.* **B298** (1993) 218.
45. M. Gronau, A. Nippe and J. L. Rosner, *Phys. Rev.* **D47** (1993) 1988; M. Gronau and J.L. Rosner, in *Proceedings of the Workshop on B Physics at Hadron Accelerators*, Snowmass, CO, June 21 - July 2, 1993, ed. P. McBride and C. S. Mishra, Fermilab Report FERMILAB-CONF-93/267 (Fermilab, Batavia, IL, 1993), p.701; *Phys. Rev.* **D49** (1994) 254.
46. C. T. Hill, in *Proceedings of the Workshop on B Physics at Hadron Accelerators*, *op. cit*, p.127; C. Quigg, *ibid.* p.443; E. Eichten, C. T. Hill and C. Quigg, *Phys. Rev. Lett.* **71** (1994) 4116.
47. S. Nussinov, *Phys. Rev. Lett.* **35** (1975) 1672.
48. M. Gronau and J. L. Rosner, *Phys. Rev. Lett.* **72** (1994) 195.
49. A. G. Cohen, D. B. Kaplan and A. E. Nelson, *Ann. Rev. Nucl. Part. Sci.* **43** (1993) 27.